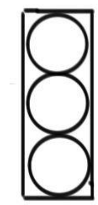
A Canister of Tennis Balls

Content Standard: 8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Standards for Mathematical Practice:

MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others**.**

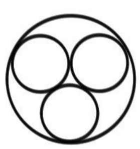
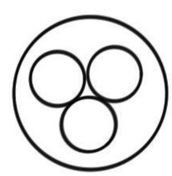
The official **diameter** of a tennis ball is 2.7 inches. They are sold in cylinder-shaped canister (Canister A) with a radius of 1.35 inches and a height of 8.1 inches. The tennis balls are stacked one on tope of each other like in the cross-section picture here.



Canister A

Answer the following questions about the tennis balls and Canister A. Give all answers in terms of Pi and round to the nearest hundredth.

1. What is the volume of the Canister A when empty?
2. What is the volume of one tennis ball?
3. In order to save money, the manufacturer would like to repackage the tennis balls into a shorter canister. He thinks this will use less space and less empty space in each canister. The packages will still be cylindrical, but the other canisters are shorter. Find the total volumes of each canister below. Advise the manufacturer which canister A, B, or C, will take up the least amount of space.

Canister B	**Canister** C
 Height = 2.7 inches Diameter = 6 inches	 Height = 2.7 inches Diameter – 8.1 inches

A Canister of Tennis Balls (Solution)

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1. Canister A height= 8.1 inches, diameter= 2.7 inches, radius= 1.35

   Use volume of cylinder $V = \pi r^2 h$

   $V = (1.35\text{ in})(1.35\text{ in})(8.1\text{ in})\ \pi$

   $V = 14.76\pi$ cubic inches

2. Tennis Ball radius = 1.35 inches

   Use Formula for volume of sphere = $4/3\ \pi\ r^3$

   $V = 4/3\ (1.35\text{ in})(1.35\text{ in})(1.35\text{ in})\ \pi$

   $= 4/3\ (2.46)\ \pi$

   $V = 3.28\ \pi$ cubic inches

3. Shorter **Canister B** volume is $24.3\ \pi$ cubic inches.
   Use volume of cylinder $V = \pi r^2 h$
   Diameter = 6 inches, radius = 3inches , height = 2.7 inches
   $V = (3\text{ in})(3\text{ in})(2.7\text{ in})\ \pi$
   Volume of short canister= $24.3\ \pi$ cubic inches

   Third short **Canister C** volume $44.29\ \pi$ cubic inches
   Use volume of cylinder $V = \pi r^2 h$
   Diameter = 8.1 inches, radius = 4.05 inches, height = 2.7 inches
   $V = (4.05\text{ in})(4.05\text{ in})(2.7\text{ in})\ \pi$
   Volume of short canister= $44.29\ \pi$ cubic inches

Volume of short **Canister B** minus the volume of tall **Canister A**: 24.3-14.76 = 9.54 π cubic inches larger.

The manufacturer is wrong because the volume for both Canister B and C are larger than Canister A, therefore they will take up more space.

The answer to the question is about comparing the volumes and explaining why the two shorter canisters have larger volumes. The height of the short Canister B will be 2.7 inches compared to the 8.1-inch height of the tall Canister A. The diameter of short Canister B is 6 inches compared to the 2.7-inch diameter of the tall Canister A. The student should consider that the diameter of B does not have to equal the height of A because the tennis balls are not arranged in a line. In the short Canister B there is some unused space between the three tennis balls in the center that has to be included in the minimum diameter making it more than two tennis balls across (2 x 2.7 plus extra space). The new short Canister B has a volume of 24.3π cubic inches. It is 9.54 π cubic inches larger in volume compared to 14.76 π cubic inches of the original Canister A. The reason being that although the tennis balls have the same volume, they require more volume for packaging due to their arrangement. A visual inspection of the drawings helps the student to infer some ideas about why the volume is larger when the 3 tennis balls are arranged differently.

The student should consider the volume of a third shorter Canister C that has the same measurements in reverse of Canister A, height 2.7 inches and diameter 8.1 inches. This would make the radius larger than the radius of the tall Canister A also. The volume of this Canister C is 44.29 π cubic inches. This choice would clearly have wasted space in packaging. The student should explore reasons why Canister A and Canister C don't have the same volume when they have reverse measurements.

The student should demonstrate an understanding of the concept of volume beyond plugging in numbers into a formula and arriving at correct answers. They should infer that the diameter influences the volume more than the height. They should also discuss why they think the volume of Canister C is so much larger than Canister A when the measurements are the same but in reverse.